Answer Set Programming with External Source Access

Reasoning Web Summer School 2017

DLVHEX

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Outline

Introduction

- Answer Set Programming (ASP): recent problem solving approach
- Term coined by DBLP:conf/iclp/Lifschitz99 [DBLP:conf/iclp/Lifschitz99,lifs-2002], proposed by others at about the same time, e.g. [Marek and Truszczyński, 1999], [Niemelä, 1999]
- It has roots in KR, logic programming, and nonmonotonic reasoning
- At an abstract level, relates to Satisfiability (SAT) solving and Constraint Programming (CP)
- Books: [Baral, 2003], [Gebser *et al.*, 2012], compact survey: [Brewka *et al.*, 2011]

Fall 2016

ANSWER SET PROGRAMMING ARTICLES

Logic Programming – Prolog

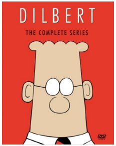
1960s/70s: Logic as a programming language (??)

- Breakthrough: Robinson's Resolution Principle (1965)

Kowalski (1979): **ALGORITHM = LOGIC + CONTROL**

- Knowledge for problem solving (LOGIC)
- "Processing" of the knowledge (CONTROL)

Prolog = "Programming in Logic"



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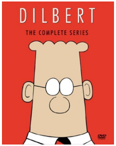
Example: Dilbert



$$man(dilbert).$$
$$person(X) \leftarrow man(X).$$
$$\text{query} \quad ?- \; person(X)$$

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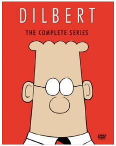

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query $?-\ person(X)$

answer $X = dilbert$

The key: techniques to search for proofs

- Proofs provide answers, based on SLD resolution
- Understanding the resolution mechanism is important
- It may make a difference which logically equivalent form is used (e.g., termination).

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Example: reverse lists

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vs

$$reverse([X|Y], Z) \leftarrow reverse(Y, U), append(U, [X], Z). \quad (2)$$

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Is this truly declarative programming?

Negation in Logic Programs

Why negation?

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Example: Dilbert cont’d

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\begin{aligned}
man(dilbert). \\
single(X) \leftarrow man(X), \mathrm{not}\, husband(X). \\
husband(X) \leftarrow fail.
\end{aligned}
$$

% fail = ”false” in Prolog

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Cyclic Negation

(cont'd)

Modifying the last rule of the Dilbert program, we obtain:

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Two intuitive models:

$$\begin{aligned} M_1 &= \{man(dilbert), single(dilbert)\}, \\ M_2 &= \{man(dilbert), husband(dilbert)\} \;\; . \end{aligned}$$

Which one to choose?

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Which one to choose? Answer set semantics: both!

LP Desiderata

Relieve the programmer from several concerns:

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“Pure” declarative programming

- Prolog does not satisfy these desiderata
- Satisfied by the *answer set semantics* of logic programs

Outline

Answer Set Programs: Syntax

Starting point: *relational signature* $S = (\mathcal{C}, \mathcal{P}, \mathcal{X})$ of pairwise disjoint sets

- $\mathcal{C}$ of *constants*,
- $\mathcal{P}$ of *predicate symbols* p/n (arity $n \geq 0$), and
- $\mathcal{X}$ of *variables*

Basic building blocks:

- *terms* are elements of $\mathcal{C} \cup \mathcal{X}$
- *atoms* are formulas $p(t_1, \ldots, t_n)$, where $p/n \in \mathcal{P}$
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Example

Typically, S is not stated explicitly if it is clear from the context; variables start with upper case letter

- terms X, bob, 123
- atoms $day()$, written as day, $firstname(bob)$, $reachable(a, Y)$
- literals $firstname(bob)$, day, $\mathrm{not}\, day$

Answer Set Programs: Syntax (cont'd)

Programs consist of rules written in "A if B" form

Rules and Programs

A *logic program* is a finite set of (disjunctive) rules r of the form

$$A_1 \vee \ldots \vee A_m \leftarrow L_1 \ldots, L_n, \quad m, n \geq 0$$

where all A_i are atoms and all L_j are literals.

- $head(r) = \{A_1, \ldots, A_m\}$ is the *head* (conclusion)
- $body(r) = \{L_1, \ldots, L_n\}$ is the *body* (premise)

Rules r with $body(r) = \emptyset$ are *facts*, and with $head(r) = \emptyset$ are *constraints*

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Example

$$day \vee night.$$
$$\leftarrow sunshine, raining.$$
$$sunshine \leftarrow day, \text{not}\, raining.$$

Safety and Recursion

Technical Requirement (by Solvers)

Each variable in a rule r must occur in $body(r)$ unnegated (*safety*).

Example

$r_1 : \ p(X) \leftarrow q(X,Y), at, \text{not}\, r(X).$ safe ✓

$r_2 : \ p(X) \leftarrow \text{not}\, t(Z).$ unsafe ×

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$$r_2 : \; p(X) \leftarrow \text{not}\, t(Z). \qquad \text{unsafe} \quad \times$$

Example: Reachability/Unreachability

$$r_1 : \quad reachable(X,Y) \leftarrow connection(X,Y).$$

$$r_2 : \quad reachable(X,Z) \leftarrow reachable(X,Y), reachable(Y,Z).$$

$$r_3 : \quad not\_reachable(X,Y) \leftarrow location(X), location(Y), \text{not}\, reachable(X,Y).$$

- Rules r_1 and r_2 express reachability (recursion)
- Rule r_3 expresses *unreachability* on top – not expressible in first-order logic!

Outline

Semantics

- Consider *ground* (i.e. *variable-free*) rules and programs
- This is lifted to arbitrary programs by variable elimination (*grounding*)

Herbrand Universe, Herbrand Base, Interpretations

Given a relational signature $S = (\mathcal{C}, \mathcal{P}, \mathcal{X})$,

- the *Herbrand universe* HU are all ground terms (i.e. $\mathcal{C}$),
- the *Herbrand base* HB is the set of all ground atoms wrt. S,
- a (Herbrand) *interpretation* is any set $I \subseteq HB$.

Intuitively, $a \in I$ means a is true in I, and false otherwise.

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Example

$P = \{ friend(X,Y) \leftarrow friend(Y,X);\ happy(X) \leftarrow friend(bob,X);\ friend(joy,bob) \}$

- $HU = \{ joy, bob \}$
- $HB = \{ friend(bob,bob), friend(bob,joy),$
  $friend(joy,bob), friend(joy,joy), happy(bob), happy(joy) \}$
- $I = \{ friend(joy,bob), friend(bob,joy), happy(joy) \}$

Semantics (cont'd)

Satisfaction of formulas, programs etc α in interpretation I, denoted $I \models \alpha$, is defined bottom up

Satisfaction, Model

An interpretation I *satisfies* (is a *model* of)

- a ground atom a, if $a \in I$;
- a literal $\text{not}\, a$, if $I \not\models a$;
- a conj. $L_1, \ldots, L_n$ of ground literals, $I \models L_i$ for $i = 1, \ldots, n$;
- a disj. $A_1 \vee \ldots \vee A_m$ of ground atoms if $I \models A_k$ for some $1 \leq k \leq m$;
- a ground rule r, if $I \models body(r)$ implies that $I \models head(r)$;
- a ground program P, if $I \models r$ for each rule $r \in P$.

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Example (cont'd)

$I = \{friend(joy, bob), friend(bob, joy), happy(joy)\}$

- $I \models happy(joy)$; $\quad I \not\models happy(bob)$
- $I \models friend(bob, joy) \leftarrow friend(joy, bob)$
- $I \models happy(joy) \vee happy(bob) \leftarrow friend(bob, joy), \mathrm{not}\, friend(joy, bob)$

Semantics (cont'd)

Example

$$P = \{\, b. \quad a \leftarrow b. \quad c \leftarrow d. \,\}$$

- $I_1 = \{b, a\}$ is a model of P
- $I_2 = \{b, a, c\}$ is a model of P as well

why should c being true in I_2 be accepted?

Semantics (cont'd)

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CWA Rationale

- Respect reit-78's [reit-78] *Closed World Assumption (CWA)*: If c is not derivable, assume it is false
- Semantically, prefer *minimal models*: a model I of P is *minimal*, if no model $J \subsetneq I$ of P exists.

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Example: CWA on mutual recursion

$$P = \{\, a \leftarrow b. \quad b \leftarrow a. \,\},$$

- $I = HB = \{a, b\}$ is a model (if P has no constraints)
- the minimal model is $I = \emptyset$

Answer Sets

Guiding Idea

- rules must be obeyed (= model)
- model must be generated by firing rules
- incorporate CWA (minimality)

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FLP-Reduct

The *FLP-reduct* P^I of a ground program P wrt. an interpretation I is obtained as follows: delete from P all rules r with false bodies:

$$P^I = \{r \in grnd(P) \mid I \models body(r)\}.$$

Answer sets of a program P are then defined as follows:

Answer Set

An interpretation I is an *answer set* of P, if I is a minimal model of P^I.

Answer Sets (cont'd)

Example: Restaurant

program P:

$$r_1: \quad restaurant(osteria).$$

$$r_2: \quad indoor(osteria) \leftarrow restaurant(osteria), \text{not}\, outdoor(osteria).$$

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  reduct $P^I = \{r_1, r_2\} = P$

- $I_2 = \{restaurant(osteria), outdoor(osteria)\}$: no answer set ×

  reduct $P^I = \{r_1\}$

Answer Sets (cont'd)

Example: Restaurant with Decision Making

$$
\begin{array}{ll}
r_1 & restaurant(osteria). \\
r_2 & indoor(osteria) \vee outdoor(osteria) \leftarrow restaurant(osteria). \\
r_3 & eat(osteria) \leftarrow indoor(osteria), raining. \\
r_4 & eat(osteria) \leftarrow outdoor(osteria), \mathrm{not}\, raining.
\end{array}
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Answer Sets (cont'd)

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answer sets:

- $I_1 = \{restaurant(osteria), indoor(osteria)\}$ ✓
  reduct $P^{I_1} = \{r_1, r_2\}$
- $I_2 = \{restaurant(osteria), outdoor(osteria), eat(osteria)\}$ ✓
  reduct $P^{I_2} = \{r_1, r_2, r_4\}$
- $I_3 = \{restaurant(osteria), indoor(osteria), raining\}$ ×
  reduct $P^{I_3} = \{r_1, r_2, r_3\}$
- all other I: ×

Non-Ground Programs

General Case: Variable Elimination (*Grounding*)

(ground) substitution: mapping $\sigma : \mathcal{X} \cup \mathcal{C} \rightarrow \mathcal{C}$ s.t. $\sigma(c) = c$ for any $c \in \mathcal{C}$

The *grounding* of (i) a rule r is $grnd(r) = \{r\sigma \mid \sigma$ is a substitution$\}$;
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Example

- P
$$reach(X,Y) \leftarrow conn(X,Y).$$
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$grnd(P) = \emptyset$ as P has no constants (in theory, let then $\mathcal{C} = \{c\}$)

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- $P' = P \cup \{\, conn(a,b). \quad conn(b,c).\, \}$

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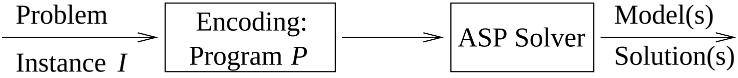
\end{array}$$

answer set $I = \{conn(a,b), conn(b,a), reach(a,b), reach(b,c), reach(a,c)\}$

ASP Paradigm

General idea: answer sets are solutions!

Reduce solving a problem instance I to computing answer sets of an LP



- Method:
  1. *encode* I as a (non-monotonic) logic program P, such that solutions of I are represented by models of P
  2. *compute* some model M of P, using an ASP solver
  3. *extract* a solution for I from M.

  variant: compute multiple/all models (for multiple/all solutions)
- Often: decompose I into *problem specification* and *data*
- Use a *guess and check* approach

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Lack of Answer Sets: Incoherence

Programs with not might lack answer sets.

Example

$$P = \{\ p \leftarrow \text{not } p.\ \}$$

NO answer set is possible ("derive p if it is not derivable")

Is this bad??

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$$barber(bertrand).$$
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- Adding $p \leftarrow q_1, \ldots, q_m, \text{not } r_1, \ldots, \text{not } r_n, \text{not } p.$

  to P, where p is fresh, "kills" all answer sets of P that (i) contain $q_1, \ldots, q_m$, and (ii) do not contain $r_1, \ldots, r_n$.
- This is equivalent to the constraint $\leftarrow q_1, \ldots, q_m, \text{not } r_1, \ldots, \text{not } r_n.$

Incomparability and Minimality

- Answer sets are minimal models of P^I.
- What about P itself?

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Proposition (Incomparability)

If I is an answer set I of a program P, then $I \models P$ and no answer set $I' \subset I$ of P exists (i.e., with $I' \subseteq I$ s.t. $I' \neq I$).

Example

- $P = \{a \leftarrow \text{not } b\}$, answer set $I = \{a\}$
- $P = \{a \leftarrow \text{not } b; \quad b \leftarrow \text{not } a; \}$, answer sets $I_1 = \{a\}$, $I_2 = \{b\}$

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In fact, answer sets satisfy a stronger property in the spirit of CWA:

Proposition (Minimality)

Every answer set I of a program P is a minimal model of P.

Non-Monotonicity

Answer sets violate the monotonicity of classical logic

Proposition (Non-monotonicity)

Given some programs P, P' and an atom a, that $I \models a$ for every answer set of P does not imply that $I \models a$ for every answer set of $P \cup P'$.

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Example: Plain Restaurant

- restaurant program P:

  $$restaurant(osteria).$$

  $$indoor(osteria) \leftarrow restaurant(osteria), \text{not}\, outdoor(osteria).$$

  answer set
  $I = \{restaurant(osteria), indoor(osteria)\} \models indoor(osteria)$

- $P \cup \{outdoor(osteria)\}$ has the answer set
  $I = \{restaurant(osteria), outdoor(osteria)\} \not\models indoor(osteria)$

Can be exploited to declare default behaviour!

Supportedness

Presence of atoms in answer sets must be supported by rules

Example

- rule $r: \ a \leftarrow b, \mathrm{not}\, c,$ model $I = \{a, b\}$
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Any answer set I of a program P is a supported model, i.e., for each $a \in I$ some rule $r \in grnd(P)$ exists s.t. $I \models body(r)$ and $I \cap head(r) = \{a\}$.

Example (cont'd)

- For $P = \{b; \quad a \leftarrow b, \text{not}\, c\}$, $I = \{a, b\}$ is an answer set
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But: stable $\neq$ minimal + supported!

Example

$$P = \{a \leftarrow a; \; a \leftarrow \mathrm{not}\; a\}$$

Computational Complexity

An answer set program P is *normal*, if each rule $r \in P$ is *normal*, defined as $|head(r)| \leq 1$.

Theorem

Deciding whether a normal program P has some answer set is

- NP*-complete in the ground (propositional) case;*
- NEXPTIME*-complete in the non-ground case.*

Theorem

Deciding whether an answer set program P has some answer set is

- Σ_2^p*-complete in the propositional case* ($\Sigma_2^p = \mathrm{NP}^{\mathrm{NP}}$);
- $\mathrm{NEXPTIME}^{\mathrm{NP}}$*-complete in the non-ground case.*

Note: the relational (i.e., function-free) non-ground case as considered here is also called *datalog case*

More on complexity: [Dantsin *et al.*, 2001]

Outline

Extensions of ASP

Language extensions like aggregates, complex formula syntax are within same semantic / computational framework

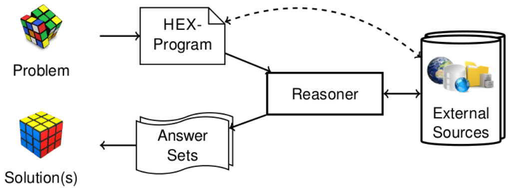
Need

- interoperability with other logics, e.g. Description Logics
- interfacing with programming languages, e.g. $C{+}{+}$, Python
- access to general *external* sources of information, e.g. WordNet

Approaches

- embedded ASP: akin to embedded SQL
- bilateral interaction: e.g. JASP
- ASP + concrete theories: constraint ASP, ASP + ontologies
- *ASP + abstract theories*: clingo, HEX/ DLVHEX

External Information Access



Examples

- import external RDF triples into the program

$$triple(S, P, O) \leftarrow \&rdf[\text{"http://}\langle Nick\rangle\text{.livejournal.com/data/foaf"}](S, P, O).$$

- access external graph

$$reachable(X) \leftarrow \&reachable[conn, a](X).$$

- perform auxiliary / data structure computations

$$fullname(Z) \leftarrow \&concat[X, Y](Z), firstname(X), lastname(Y).$$

External Information Access (cont'd)

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    $\Rightarrow$ no simple fixpoint computation

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- **Formal Model of External Atoms**
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  - allow arbitrary external code
    $\Rightarrow$ "impedance mismatch"
- **Semantics**
  - e.g. cyclic reference (web graphs!)
  - non-monotonic external sources
    $\Rightarrow$ no simple fixpoint computation
- **Value Invention**
  - new ground terms might appear

Outline

Syntax

New element: $\mathcal{G}$ external predicate names $\&g$ that have $in(\&g)$ many "input" arguments and $out(\&g)$ many "output" arguments

External Atom

An *external atom* over a rel. signature $S = (\mathcal{C}, \mathcal{P}, \mathcal{X}, \mathcal{G})$ is of the form

$$\&g[Y_1, \ldots, Y_n](X_1, \ldots, X_m)$$

where

- $Y_1, \ldots, Y_n$ are terms and predicate names from $\mathcal{C} \cup \mathcal{X} \cup \mathcal{P}$ (*input list*)
- $X_1, \ldots, X_m$ are terms from $\mathcal{C} \cup \mathcal{X}$ (*output list*)
- $\&g \in \mathcal{G}$ is an external predicate name with $in(\&g) = n$, $out(\&g) = m$

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Examples

- $\&rdf[U](S, P, O)$: intuitively, from a given concrete "input" URL U (a constant), retrieve (one by one) all "output" triples (S, P, O)
- $\&reachable[connection, a](X)$: intuitively, all nodes X reachable from node a in a graph represented by atoms of form $connection(u, v)$.

External Atoms

Examples (cont'd)

- $\&concat[X, Y](Z)$: intuitively, concatenate two strings
  - $\&concat[bob, dylan](bobdylan)$ is true
  - $\&concat[bob, dylan](Z)$ is true for $Z = bobdylan$
  - $\&concat[bob, Y](bobdylan)$ is true for $Y = dylan$

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External atoms can be of any nature (non-logical) nature

Example

$$\&weatherreport[dateLocationPredicate](WeatherConditions)$$

query a web-based weather report

- input $dateLocationPredicate$ is a binary predicate with tuples (d,l) of dates d and locations l (facts $dateLocationPredicate(d,l)$)
- output $WeatherConditions$ are (one by one) all weather conditions that occur at some input date & location

$\&weatherreport[goto](W)$ where $goto = \{(1,paris),(1,london),(2,paris),(2,london)\}$ returns all weather conditions on dates 1/2 for London/Paris

HEX Programs

HEX rule and program

A HEX *program* is a set P of *(HEX) rules* r of the form

$$A_1 \vee \ldots \vee A_m \leftarrow L_1 \ldots, L_n, \quad m, n \geq 0,$$

where all A_i are atoms, and all L_j are either literals or HEX-literals, i.e. either

- an ordinary literal,
- an external atom,
- or a default-negated external atom.

That is, like ordinary ASP rules/programs but external atoms can occur in rule bodies

Examples

- $reachable(X) \leftarrow \&reachable[connection, a](X).$
- $fullname(Z) \leftarrow \&concat[X, Y](Z), firstname(X), lastname(Y).$
- $\leftarrow \&weatherreport[goto](W), badweather(W).$

HEX Programs (cont'd)

Example: City Trip

Plan to visit Paris and London, under the condition the weather isn't bad

Program Π_{goto}:

$$
\begin{array}{lr}
r_1 & badweather(rain). \quad badweather(snow). \\
r_2 & goto(1, paris) \vee goto(1, london). \\
r_3 & goto(2, paris) \vee goto(2, london). \\
r_4 & \leftarrow \&weatherreport[goto](W), badweather(W).
\end{array}
$$

- state what bad weather means (r_1)
- decide on what day to go to which city (r_2, r_3)
- exclude trips where the (external) weather report indicates bad weather during the trip (r_4)

Outline

Semantics

Analogous to ordinary ASP:

- the *Herbrand base* HB for HEX program P
- the *grounding* of a rule r, $grnd(r)$, and of P, $grnd(P) = \bigcup_{r \in P} grnd(r)$.
- *interpretations* are subsets $I \subseteq HB$ with no external atoms

To define satisfaction, key issue is the semantics of external atoms.

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To define satisfaction, key issue is the semantics of external atoms.

Oracle Function

Every $\&g \in \mathcal{G}$, has an associated decidable *oracle function*

$$f_{\&g} : 2^{HB_P} \times (\mathcal{C} \cup \mathcal{P})^n \times \mathcal{C}^m \to \{\mathbf{T}, \mathbf{F}\}, \quad n = in(\&g), m = out(\&g)$$

that maps each $(I, \vec{y}, \vec{x})$, where $I \subseteq HB$ is an interpretation, $\vec{y} = y_1, \ldots, y_n$ on $\mathcal{C} \cup \mathcal{P}$ is "input", and $\vec{x} = x_1, \ldots, x_m$ on $\mathcal{C}$ is "output", to $\mathbf{T}$ or $\mathbf{F}$.

Pragmatic assumptions:

- for any $I, \vec{y}$, only finitely many $\vec{x}$ yield $f_{\&g}(I, \vec{y}, \vec{x}) = \mathbf{T}$
- output $\vec{x}$ is independent of the extensions of the predicates that do not occur in the input $\vec{y}$

Oracle Functions

Example: String Concatenation

for the external predicate $\&concat$, the associated function is

$$f_{\&concat}(I, X, Y, Z) = \begin{cases} \mathbf{T}, & \text{if } XY = Z; \\ \mathbf{F}, & \text{otherwise} \end{cases}$$

(where XY is concatenation of X and Y)

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(where XY is concatenation of X and Y)

Example: City Trip (cont'd)

- weather forecast Paris: sun on day 1 and day 2
- weather forecast London: rain on day 1 and day 2

the corresponding oracle function is ($wr = weatherreport$)

$$f_{\&wr}(I, goto, W) = \begin{cases} \mathbf{T}, \text{ if } \{goto(1, london), goto(2, london)\} \subseteq I \text{ and } W = rain, \\ \mathbf{T}, \text{ if } \{goto(1, london), goto(2, paris)\} \subseteq I \text{ and } W \in \{sun,\ rain\}, \\ \mathbf{T}, \text{ if } \{goto(1, paris), goto(2, london)\} \subseteq I \text{ and } W \in \{sun,\ rain\}, \\ \mathbf{T}, \text{ if } \{goto(1, paris), goto(2, paris)\} \subseteq I \text{ and } W = sun, \\ \mathbf{F}, \text{ otherwise.} \end{cases}$$

Satisfaction and Models

Satisfaction of External Atom

An interpretation $I \subseteq HB$ *satisfies* (is a *model* of) a ground external atom $a = \&g[\vec{y}](\vec{x})$, denoted $I \models a$, if $f_{\&g}(I, \vec{y}, \vec{x}) = \mathbf{T}$.

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- $I \models \&concat[bob, dylan](bobdylan)$ holds for every interpretation I
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Example: City Trip (cont'd)

For weather forecast as above:

- $I \models \&weatherreport[goto](sun)$ holds if $I \models goto(1, paris)$, or if $I \models goto(2, paris)$.
- $I \models \&weatherreport[goto](rain)$ if $I \models goto(1, london)$ or if $I \models goto(2, london)$,

Answer Sets for HEX Programs

Answer sets naturally extend to HEX-programs

Answer Set of a HEX Program

An interpretation $I \subseteq HB$ is an *answer set* of a HEX program P, if I is a minimal model of the *FLP-reduct*

$$P^I = \{r \in grnd(P) \mid I \models body(r)\}.$$

$AS(P)$ = the set of all answer sets of P

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Remarks:

- For ordinary P (no external atoms), the answer sets are as usual
- For aggregates modeled as external atoms (e.g. $\&count[goto](N)$), the answer sets coincide with FLP-answer sets [Faber *et al.*, 2011]
- Alternative (more restrictive) notions of answer sets exist [Shen *et al.*, 2014]

Answer Sets for HEX Programs

Example: City Trip (cont'd)

$$
\begin{array}{ll}
\Pi_{goto} & badweather(rain). \quad badweather(snow). \\
& goto(1, paris) \vee goto(1, london). \\
& goto(2, paris) \vee goto(2, london). \\
& \leftarrow \&weatherreport[goto](W), badweather(W).
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$$

- For the above weather report, Π_{goto} has one answer set: $\{goto(1, paris), goto(2, paris), badweather(snow), badweather(rain)\}$

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- For a different weather report saying it's always sunny, 3 more answer sets exist:
  - $\{goto(1, paris), goto(2, london), badweather(snow), badweather(rain)\}$
  - $\{goto(1, london), goto(2, paris), badweather(snow), badweather(rain)\}$
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Answer Sets for HEX Programs

Example: City Trip (cont'd)

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$$goto(1, paris) \vee goto(1, london).$$
$$goto(2, paris) \vee goto(2, london).$$
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  - $\{goto(1, london), goto(2, london), badweather(snow), badweather(rain)\}$
- Finally if the weather report for both cities is *snow* for days 1 and 2, no answer set exists.

Outline

Basic Properties

The basic properties of answer sets extend to HEX-programs:

- answer sets are incomparable
- answer sets are minimal models
- answer sets are supported models
- non-monotonicity

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The computational complexity depends on external atoms: deciding answer set existence is

- Σ_2^p-complete for ground programs, if evaluating external atoms, i.e. deciding whether $f_{\&g}(I, \vec{y}, \vec{x}) = \mathbf{T}$ holds, is feasible in polynomial time with an NP oracle;
- Σ_2^p-hard already for Horn ground programs (no disjunction, no negation) and polynomial-time external atoms.
- Thus, *minimality checking of answer set candidates for* HEX*-programs is a challenging problem*

Outline

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Modeling techniques from ordinary ASP carry over to HEX-programs.

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Example: 3-Colorability of a Graph

Consider a graph $G = (V, E)$
given by facts $node(v)$ for all $v \in V$ and $edge(u, v)$ for all $(u, v) \in E$.

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&\leftarrow r(X), r(Y), edge(X, Y) \\
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\end{aligned}
$$

Basic Methodology (cont'd)

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   - design a program P and an answer set candidate I_{sat} such that I_{sat} is the single answer set of P if the property Pr holds, and
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Example: Non-3-Colorability of a Graph

$$
\begin{aligned}
b(X) \vee r(X) \vee g(X) &\leftarrow node(X)\\
non\_col &\leftarrow r(X), r(Y), edge(X,Y)\\
non\_col &\leftarrow g(X), g(Y), edge(X,Y)\\
non\_col &\leftarrow b(X), b(Y), edge(X,Y)\\
r(X) &\leftarrow non\_col, node(X)\\
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$$
\begin{aligned}
r(X) \vee g(X) \vee b(X) &\leftarrow node(X) \\
&\leftarrow \text{not } \&check[edge, r, g, b]()
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- External sources may combine both use cases.
- Important: Both usages are based on the same language features!

Computation Outsourcing

On-demand Constrains

- Constraints of form
  $$\leftarrow \&forbidden[p_1, \ldots, p_n]()$$
  eliminate certain extensions of predicates $p_1, \ldots, p_n$.

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- Example:
Efficient planning in robotics where external atoms verify the feasibility of a 3D motion [Erdem *et al.*, 2016b].

Computation Outsourcing (cont'd)

Accessing Procedural Computations

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Complexity Lifting

- Computations with a complexity higher than the complexity of ordinary ASP programs.
- External sources can also be other ASP or HEX programs, which allows for encoding other formalisms of higher complexity in HEX programs, e.g., *abstract argumentation frameworks* [Dung, 1995].

Information Outsourcing

Data Sources

- RDF triplet stores:
  $p(X, Y) \leftarrow url(U), \&rdf[U](X, Y, Z)$
- Geographic data
- Description logic ontologies
- Multi-context systems
- Relational databases

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Note:
Some external sources may realize a combination of data and computation outsourcing (e.g. complex queries over ontologies).

Outline

Modeling an Application

How to realize an application on top of HEX-programs?

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These steps might be repeated or interleaved.

External atoms might be reused for multiple applications.

Outline

Applications of HEX-Programs

Some examples:

- Queries of Web resources (RDF triplet stores, social graphs, etc)
- Multi-context Systems (interconnection of knowledge-bases)
- DL-programs (integration of ASP with ontologies)
- Constraint ASP (programs with constraint atoms)
- Physics simulation (e.g. AngryBirds agent)
- Route planning (possibly semantically enriched)
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- ACTHEX (programs with action atoms)

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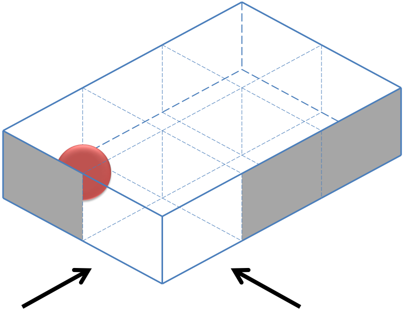

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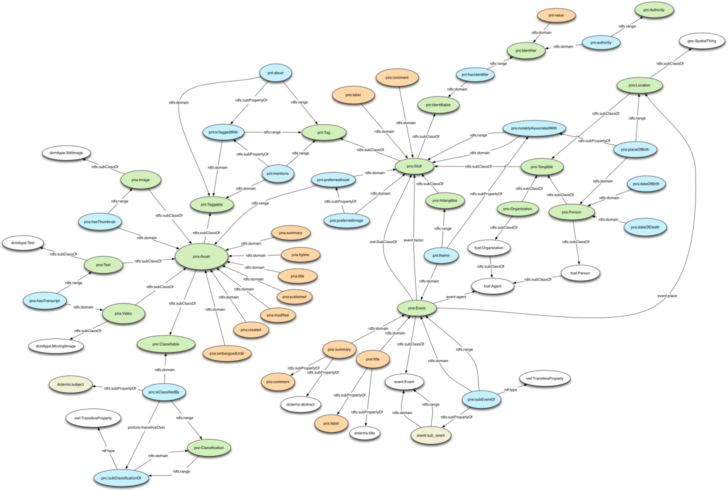

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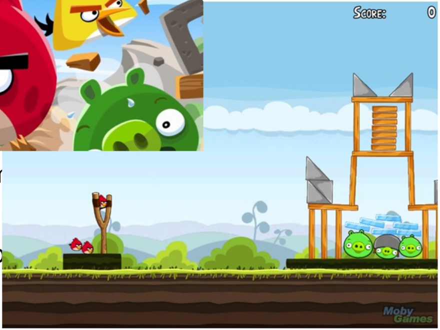

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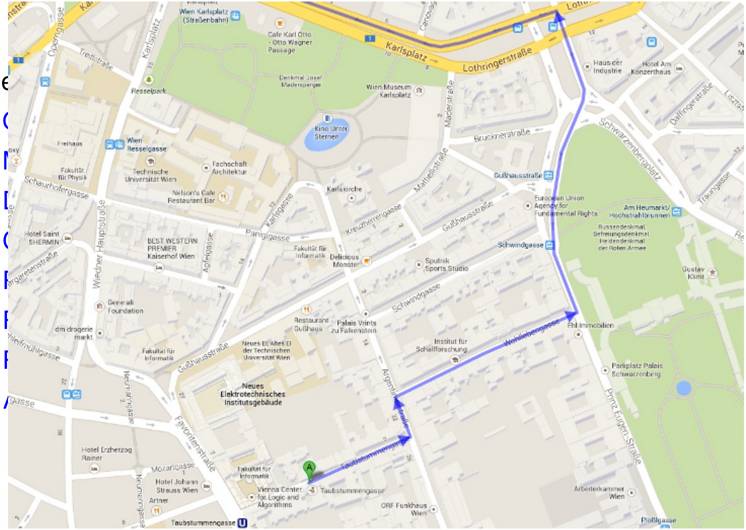

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Example: Semantic Web Application

Example: Friend-of-a-Friend

Use the FOAF (Friend-of-a-friend) RDF schema to return all pairs of nicknames that know each other, as stored in a FOAF RDF datasource:

$$explore(\text{"http://}\langle Nick\rangle\text{.livejournal.com/data/foaf"})$$

$$triple(S,P,O) \leftarrow \&rdf[What](S,P,O),\ explore(What)$$

$$\begin{aligned} knows(Nick_1, Nick_2) \leftarrow\ & triple(Id_1, \text{"http://xmlns.com/foaf/0.1/knows"}, Id_2), \\ & triple(Id_1, \text{"http://xmlns.com/foaf/0.1/nick"}, Nick_1),\ Nick_1 < Nick_2, \\ & triple(Id_2, \text{"http://xmlns.com/foaf/0.1/nick"}, Nick_2). \end{aligned}$$

$$knows(A,C) \leftarrow knows(A,B),\ knows(B,C)$$

Example: Semantic Web Application (cont'd)

Example: Recursive FOAF querying with limited depth

$explore($"http://$\langle Nick \rangle$.livejournal.com/data/foaf"$)$

$explore\_to(What, 3) \leftarrow explore(What)$

$triple\_at(S, P, O, D) \leftarrow \&rdf[Uri](S, P, O),\ explore\_to(Uri, D),\ D > 1$

$explore\_to(U, D_2) \leftarrow D_2 = D_1 - 1,$

$\quad triple\_at(Id,$ "http://www.w3.org/2000/01/rdf-schema#seeAlso"$, U, D_1),$

$\quad triple\_at(Id,$ "http://xmlns.com/foaf/0.1/nick"$, Nick, D_1)$

$found(Nick) \leftarrow triple\_at(S,$ "http://xmlns.com/foaf/0.1/nick"$, Nick, D).$

Example: Physics Simulation

Example: AngryHEX

Fundamental strategy:
Maximize the estimated damage to obstacles and pigs.

$$
\begin{aligned}
shootable(O, Type, Tr) \leftarrow\ & \&shootable[O, Tr, V, Sx, Sy, Sw, Sh, B, bb](O), \\
& birdType(B),\ velocity(V),\ objectType(O, Type), \\
& slingshot(Sx, Sy, Sw, Sh),\ trajectory(Tr) \\
tgt(O, Tr) \vee ntgt(O, Tr) \leftarrow\ & shootable(O, Type, Tr) \\
\leftarrow\ & target(X, \_),\ target(Y, \_), X \neq Y. \\
\leftarrow\ & target(\_, T_1),\ target(\_, T_2),\ T_1 \neq T_2 \\
target\_ex \leftarrow\ & target(\_, \_) \\
\leftarrow\ & \text{not}\ target\_ex. \\
directDmg(O, P, E) \leftarrow\ & target(O, Tr),\ objectType(O, T),\ birdType(Bird), \\
& dmgProbability(Bird, T, P), \\
& energyLoss(Bird, T, E)
\end{aligned}
$$

Example: Physics Simulation

Example: AngryHEX (cont'd)

$$
\begin{aligned}
exDirectDmg(O) &\leftarrow directDmg(O, \_, \_) \\
nexDirectDmg(O) &\leftarrow \text{not } exDirectDmg(O),\ objectType(O, \_) \\
goodObject(O) &\leftarrow objectType(O, pig) \\
goodObject(O) &\leftarrow objectType(O, tnt) \\
&\leftsquigarrow nexDirectDmg(O), goodObject(O) \quad [1@4, O, nexDirectDmg] \\
&\leftsquigarrow nexDirectDmg(O). \quad [1@1, O, nexDirectDmg]
\end{aligned}
$$

The DLVHEX-System

DLVHEX

`http://www.kr.tuwien.ac.at/research/systems/dlvhex`

- Based on GRINGO and CLASP from the Potassco suite.
- Supported platforms: Linux-based, OS X, Windows.
  Pre-compiled binaries available.
- External sources are implemented as plugins using a plugin API (available for C++ or Python).
- Support for the ASP-Core-2 standard.
- Online demo:
  `http://www.kr.tuwien.ac.at/research/systems/dlvhex/demo.php`.
- User manual available (see system website).

System Architecture

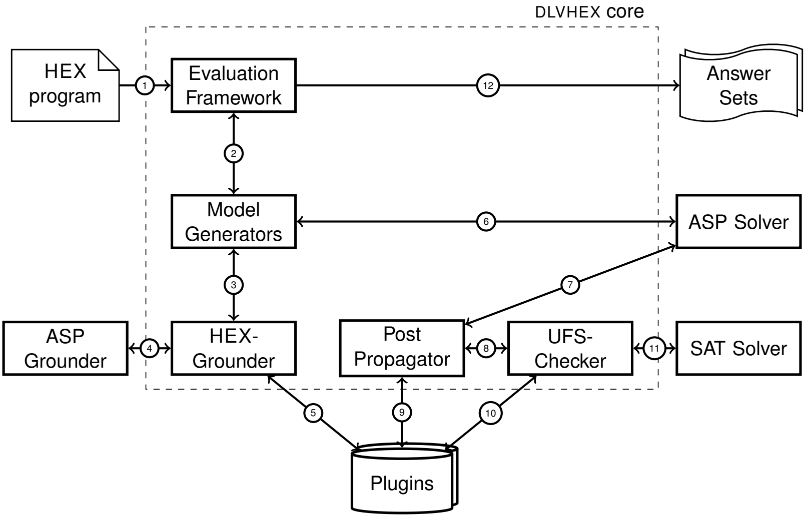

Figure: Architecture of DLVHEX

Outline

Python Programming Interface

More convenient interface

Previously only C++ support, but Python preferred by many developers:

- No overhead due to makefiles, compilation, linking, etc.
- High-level features.
- Negligible overhead compared to plugins implemented in C++.

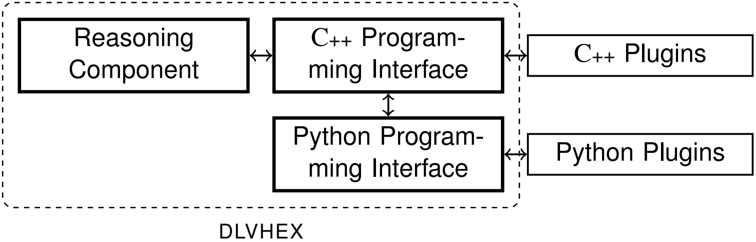

Figure: Architecture of the Python Programming Interface

Python Programming Interface (cont'd)

Example

Program

$$\Pi = \left\{ \begin{array}{l} r_1: start(s). \\ r_2: reach(X) \leftarrow start(X). \quad r_3: reach(Y) \leftarrow reach(X), \&edge[X](Y). \end{array} \right\}$$

compute the nodes reachable from a start node s in a graph.

Implementation of $\&edge[X](Y)$:

```
def edge(x):
  graph=((1,2),(1,3),(2,3))          # simplified implementation
  for edge in graph:                 # search for out-edges of node x
    if edge[0]==x.intValue():
      dlvhex.output((edge[1],))      # output edge target

def register():
  prop = dlvhex.ExtSourceProperties() # inform dlvhex about
  prop.addFiniteOutputDomain(0)       # finiteness of the graph
  dlvhex.addAtom("edge", (dlvhex.CONSTANT, ), 1, prop)
```

Outline

From Black-box to Grey-box

Overcoming the Evaluation Bottleneck

- By default, external sources are seen as black boxes.
- Behavior under an interpretation does not allow for drawing conclusions about other interpretations.
- Algorithmic improvements require meta-information about external sources.

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Idea

- Developers of external sources and/or implementer of HEX-program might have useful additional information.
- Provide a (predefined) list of possible properties of external sources.
- Let the developer and/or user specify which properties are satisfied.
- Algorithms exploit them for various purposes, most importantly:
  - efficiency improvements and
  - language flexibility (reducing syntactic restrictions).

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Important:
User specifies them but does **not** need to know how they are exploited!

Specifying Properties

Available properties (examples)

- **Functionality**: $\&add[X, Y](Z)\langle functional\rangle$
  Adds integers X and Y and is true for their sum Z.
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  Example:
  $\&greaterThan[p, 10]()$ is true if $\sum_{p(c)\in I} c > 10$.
  It is monotonic for positive integers.

Exploiting Properties for Efficiency Improvement

Conflict-driven Solving

- ASP program is internally represented by nogoods (sets of literals which cannot be simultaneously true).
- Additional nogoods learned from conflicting interpretations.
- HEX-solver further learns nogoods from external sources which describe parts of their behavior to avoid future wrong guesses.

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- We evaluate $\&\mathit{diff}[p,q](X)$ under $I = \{p(a), q(b)\}$.
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Exploiting Properties

- Known properties used to shrink nogoods to their essential part.
- Example: $\&\mathit{diff}[p,q](X)$ is monotonic in p:
  Shrink above nogood N to $N' = \{p(a), \neg q(a), q(b), \neg \&\mathit{diff}[p,q](a)\}$.
  (If $p(b)$ turns to true, $\&\mathit{diff}[p,q](a)$ is still true $\Rightarrow \neg p(b)$ not needed.)

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Solution: Syntactic Restrictions (Safety)

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- But: overly restrictive.

Exploiting Properties

- Properties may allow for identifying finite groundability even in presence of recursive value invention (in some cases).
- Example:
  Known finiteness of the graph above allows for establishing safety.

Outline

Use Case: Semantic Trip Planning in Vienna

Requirements

- Find shortest trip visiting predefined locations
- Long trip ⇒ add lunch location using an ontology
- Choose restaurant depending on weather report

DEMO

Trip Planning

- Transport data might be:
  - Extremely large
  - Remote/not accessible
- Access external transport information (information outsourcing)
- Use dedicated algorithm to compute shortest connection (computation outsourcing)

External atom:
`&route[File,Loc1,Loc2](Stp1,Stp2,Costs,Line)`

$\Rightarrow$ Obtain shortest trip by using weak constraints

DEMO

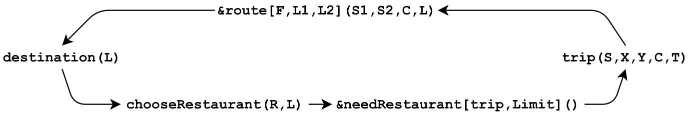
Adding Lunch Location

- Adjustment of the trip based on its length
- Add on-demand constraint (no output needed)
- Boolean output depends monotonically on the input
  - Specify according property

External atom:

`&needRestaurant[trip,Limit]()`

Introduces cyclic dependency, not strongly safe:



DEMO

Partial Evaluation

- `&needRestaurant[trip,Limit]()` usually evaluated only after extension of `trip` is decided
  - Truth value not fixed before
- Often truth value can be decided early during search
- Partial assignments: atoms can be *true*, *false* or *unassigned*
- Use both methods isTrue() and isFalse()
  - Everything else is unassigned
- Use both methods output() and outputUnknown() to declare outputs
  - All other outputs are implicitly false
- Requirement: assignment monotonicity

Example

Learned nogood: $\{\neg t(0,1), t(1,1), t(2,1), t(3,1), \&nR[t,3]()\}$

DEMO

DL-Lite Plugin

- We use the DL-Lite Plugin for semantically enriched route planning (inspired by [Eiter *et al.*, 2016c])
- Interfaces to OWL ontologies using DL reasoner
- Provides external atoms for concept and role queries:
  - `&cDL[File,rp,rm,cp,cm,C](X)`
  - `&rDL[File,rp,rm,cp,cm,R](X,Y)`
- Bidirectional interaction by adding elements to concepts and roles, resp. to their complements

Link:
http://www.kr.tuwien.ac.at/research/systems/dlvhex/dlliteplugin.html

Restaurant Ontology

$$
\begin{aligned}
BeerGarden &\sqsubseteq Restaurant \\
BeerGarden &\sqsubseteq \neg IndoorRestaurant \\
IndoorRestaurant &\sqsubseteq Restaurant \\
IndoorRestaurant &\sqsubseteq \neg BeerGarden \\
IndoorRestaurant &\sqsubseteq \neg WurstStand \\
Restaurant &\sqsubseteq \exists closeTo.Location \\
WurstStand &\sqsubseteq Restaurant \\
WurstStand &\sqsubseteq \neg IndoorRestaurant
\end{aligned}
$$

$$
\begin{aligned}
&Location(Karlsplatz) \\
&Location(Museumsquartier) \\
&Location(Praterstern) \\
&BeerGarden(bg1) \\
&closeTo(bg1, Praterstern) \\
&IndoorRestaurant(ir1) \\
&closeTo(ir1, Museumsquartier) \\
&WurstStand(ws1) \\
&closeTo(ws1, Karlsplatz)
\end{aligned}
$$

DEMO

Weather Data

- Goal: retrieve weather data from http://openweathermap.org/
- Importing dynamic data from remote location
- General plugin for retrieving JSON data from API
  - Data represented by nested key-value pairs:
    `{"weather":[{"id":803,"main":"Clouds",`
    `"description":"clouds", "icon":"04d"}], ...}`
- Input type `dlvhex.TUPLE` for arbitrary number of constants
  - Provide sequence of keys

External atom:

`&getJSON[Url,Keys.TUPLE](Value)`

DEMO

Summary of the Case Study

- Encoding uses four different external atoms in combination
  - `&route`-Plugin for information and computation outsourcing
  - `&needRestaurant`-Plugin for external check
  - DL-Lite-Plugin for interfacing an external DL-reasoner
  - `&getJson`-Plugin for accessing remote information on the web
- Complete implementation and more examples at:
  https://github.com/hexhex/manual/tree/master/RW2017/

Outline

HEX$^{\exists}$ Programs

- By value invention external atoms can generate witnesses
- Used to model query answering from existential rules

Example

Not possible in standard ASP:

$$\exists X\colon \mathit{office}(Y,X) \leftarrow \mathit{employee}(Y).$$

Encoding with external atom:

$$\mathit{office}(Y,X) \leftarrow \mathit{employee}(Y), \&\mathit{exists}[r_1, Y](X).$$

HEX Programs with Function Symbols

- External atoms can simulate composition and decomposition of function terms
- Allows external data type checking and argument generation

Example

Not possible in standard ASP:

$$
\begin{aligned}
q(f(X)) &\leftarrow p(X). \\
r(Y) &\leftarrow q(f(Y)).
\end{aligned}
$$

Encoding with external atom:

$$
\begin{aligned}
q(A) &\leftarrow p(X), \&comp[f, X](A). \\
r(Y) &\leftarrow q(B), \&decomp[B](f, Y).
\end{aligned}
$$

ACTHEX

- Extension of HEX for execution of declaratively scheduled actions
- Action atoms in rule heads operate on an external environment
- Environment can influence truth values of external atoms
  - Enables stateful behaviour

Example

$$
\begin{aligned}
\#robot[clean, kitchen]\{c, 2\} &\leftarrow night \\
\#robot[clean, bedroom]\{c, 2\} &\leftarrow day \\
\#robot[goto, charger]\{b, 1\} &\leftarrow \&sensor[bat](low) \\
night \vee day &\leftarrow
\end{aligned}
$$

Constraint HEX Programs

- Grounding issues when encoding constraints in ASP
- Contain ordinary, external and constraint atoms
  - Comparisons of arithmetic expressions
- Allow to combine diverse background theories

Example

$$food(P) \leftarrow \&sql[\text{“Select price from Food”}](P)$$
$$drink(P) \leftarrow \&sql[\text{“Select price from Drink”}](P)$$
$$inMenu(F,D) \vee outMenu(F,D) \leftarrow food(F), drink(D)$$
$$F + D < P \leftarrow inMenu(F,D), max\_price(P)$$

Encoding of constraint with external atom:

$$con(F,+,D,<,P) \vee con(F,+,D,\geq,P) \leftarrow inMenu(F,D), max\_price(P)$$
$$\leftarrow \text{not } \&check[con]()$$

Nested HEX [Eiter *et al.*, 2013]

- External atoms for evaluating subprograms and inspecting their answer sets:
  $\&callhex$, $\&callhexfile$, $\&answersets$, $\&predicates$, $\&arguments$
- A new instance of DLVHEX is called and results stored in an answer cache assigning unique handles

Example

$$
\begin{aligned}
p_1(x,y) &\leftarrow \\
p_2(a) &\leftarrow \\
p_2(b) &\leftarrow \\
handle(PH) &\leftarrow \&callhexfile[\texttt{"sub.hex"}, p_1, p_2](PH) \\
ash(PH, AH) &\leftarrow \&callhex[\texttt{"a v b :-"}](PH), \&answersets[PH](AH)
\end{aligned}
$$

Outline

Related Work

- Many approaches, different degrees of integration
- DLV^{DB} offers access to relational databases via *ODBC* interface
- ONTODLV for information retrieval from OWL ontologies, extends ASP with classes, inheritance, relations and axioms
- DLV-EX programs early generic integration approach
  - Introduction of new terms by value invention
  - Only terms as inputs to external sources
  - Nonmonotonic aggregates not expressible
- CLINGO supports custom functions implemented in *Lua* or *Python*
  - Import extensions of user-defined predicates during grounding
  - Customisable built-in atoms
  - No cyclic dependencies

Related Work (cont'd)

- CLINGO 5 provides generic interfaces for theory solving in ASP
  - Semantics differs from HEX unfounded support of theory atoms allowed $\Rightarrow$ consider $p \leftarrow \&id[p]()$
  - Theory atoms interrelated via external theory (orthogonal to HEX)
  - No value invention based on answer set
  - Well-suited for system developers, rich infrastructure
- Extensions of ASP with specific external sources:
  - Constraint ASP solvers, e.g. CLINGCON, lc2casp, ezcsp, EZSMT
  - Extensions of ASP with SMT, e.g. dingo (difference logic), ASPMT

Outline

Summary

- HEX is a powerful formalism, wide range of applications
- Extends ASP with external sources via API-style interface
- Bi-directional interaction and value invention possible
- Methodology from ASP generalises to HEX
- Implemented in the DLVHEX system
  - Plugins in Python and C++
  - Exploiting external source properties

DLVHEX

http://www.kr.tuwien.ac.at/research/systems/dlvhex/

Outline

Further Resources

- All executable examples from this course:
  `https://github.com/hexhex/manual/tree/master/RW2017/`
- Slides of tutorial "ASP for the Semantic Web" and many executable ASP/HEX-examples:
  `http://asptut.gibbi.com/`
- An online demo of the DLVHEX system:
  `http://www.kr.tuwien.ac.at/research/systems/dlvhex/demo.php`
- Pre-built binaries of DLVHEX for Linux, OS X and Windows:
  `http://www.kr.tuwien.ac.at/research/systems/dlvhex/downloadb.html`
- The source code of DLVHEX and corresponding plugins, best place for bug reports:
  `https://github.com/hexhex/`
- Python-based HEX implementation for a fragment of the HEX language and a subset of features
  `https://github.com/hexhex/hexlite`

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